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Synthetic Dimensionality

THE RECURSIVE ALGEBRA OF SEMANTIC SPACE

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UNITED COMMUNITIES OF SPIRIT

"Nobody can say what a variable is."
Hermann Weyl

Abstract

SYNTHETIC dimensionality is an algebraic model of class and category structure based on the concept of dimension. In this model, actual objects are quantitatively described in terms of values in n dimensions. A class of objects is defined in these dimensions, where the similarities or common properties of the objects are defined in dimensional values, as are their distinguishing differences.

If the objects of a class can be (sequentially) ordered by their values in their defining dimensions, this class is itself algebraically isomorphic to a dimension, and the objects (elements, members) of the class can be thought of as values of the dimension. A "synthetic" dimension is thus defined as an ordered class of abstract objects (elements, members) themselves described in more than one simultaneous dimension/value.

The recursive properties of this approach to class specification (ie, dimensions are defined in terms of dimensions) appears to lead to a surprising and elegantly simple characterization of semantic space. This paper introduces the basic ideas of this method, and argues that a synthetic dimension is a "universal primitive" from which all conceptual structure can be assembled.

Contents

1. Introduction and Project History

IN 1987, equipped with a small personal computer and a hypertext-type outline processor, I began organizing and collating a glossary of epistemological concepts and definitions, attempting to build a general model of cognitive or semantic space. I was guided by my basic instinct that conceptual structure is generally organized across a "hierarchy of abstraction", and in previous years I had explored the issues involved in the algebraic representation of this structure. In these terms, for example, there is a nested hierarchical relationship between the concepts of "furniture", "chair" and "rocking chair". Clearly, a chair is a type of furniture, and a rocking chair is a type of chair. These three classes of object are thus related to one another across a descending series of levels of abstraction, which might have at its bottom level a particular actual rocking chair. My guiding instinct was that the entire conceptual structure of cognition is more or less organized in similar terms.

But my attempts to analytically map the whole of cognitive space in terms of this hierarchical structure initially met with little success. I had drawn hundreds of tree-structured diagrams that characterized this relationship, and I was especially interested in "mandala-like" diagrams that modeled these relationships over a series of redifferentiated concentric circles. These diagrams modeled (defined) "absolute unity" and "the highest level of abstraction" at the center of these concentric circles, and the infinite multiplicity and diversity of particular things and objects as distributed at the periphery. I found this diagram highly intuitive, and consistent with many schools of psychology. But try as I might, I could not find a useful or reliably accurate way to map or describe what was apparently a highly "multi-dimensional" set of relationships through this diagram. Multi-faceted abstract objects simply could not be classified or interpreted "the same way every time".

Any two-dimensional tree-structured diagram drawn on paper suffers from the rigidity of what has been called an "Aristotelian type hierarchy". A clear example of this limitation can be found in a university library, where each book has to occupy only one exact shelf position, addressed through the Dewey Decimal System. In today's scientific environment, where the most interesting subjects often involve the intersection of several traditional academic disciplines or departments, a particular book might be categorized in any number of alternative ways, but can only occupy one shelf position. Today, librarians can overcome this limitation by searching for the book electronically through any number of relevant descriptive keywords.

Though I failed in my initial attempts to diagram the hierarchical relationship between abstract and empirical concepts, I was persuaded that this fundamental principle, highly intuitive and natural, is not wrong. As I saw it, the best and most desirable answer to this difficult problem does not lie in so-called "semantic networks", which tend to be arbitrary heuristics, but in terms of a flexible and adaptive general hierarchy, which is algebraically defined so as to satisfy the many simultaneous constraints governing the linkage of concepts across levels of abstraction.

This is an enormously subtle and challenging algebraic problem, which has yet to be overcome by researchers working in this field. As I see it, the problem is akin to a mysterious and fabulously complex lock for which we as yet have no key -- and I believe that the only hope of opening this lock, and meeting these simultaneous constraints, is a relentless trial-and-error approach that considers hundreds of alternatives and is guided by deep intuition. Since we are searching for a general interpretive model or set of rules, which will be applicable to all instances throughout semantic space, the opening of this lock must be pursued "from the top down", and no key will work consistently until every detail is perfect. Having pursued this trial and error approach for several years, it is currently my sense that the recursive methods of synthetic dimensionality do provide a means to overcome this problem. By defining the elements of semantic space in terms of abstract dimensions, I believe it is now possible to elegantly integrate the otherwise overwhelming complexity of conceptual structure and terminological diversity into a single set of general principles.

1.1 The Epistemological Dictionary

AS I continued to work with my outline processor, I built a vocabulary of approximately 300 fundamental epistemological concepts, which I attempted to systematically define in consistent algebraic terms, free-associating for hours at a stretch as I worked with these ideas. I was deeply involved with the concept of "information", and the digital bit-structure of symbolic representation. For me, a concept was an "information structure" (as per Peter Wegner's 1968 classic *Programming Languages, Information Structures, and Machine Organization*), and I wanted to map or diagram the meaning of that structure across its levels of abstraction.

Going over and over this material, from many alternative points of view, and factoring in the most fertile scientific literature I could find, one single idea began to appear over and over again: the concept of dimension. I intuitively felt that all abstract categories or concepts could be linearly factored into their constituent dimensionality, and that the meaning of an abstract concept could be defined in terms of a (adaptive and ad hoc) cascade of dimensions usually grounded in the "empirical dimensionality" of quantitative measurement. Clearly, when we wish to describe an actual object exactly, we describe it in terms of quantitative measurement. But how do we descriptively characterize abstractions in terms of dimensions?

We might, for example, distinguish between various kinds of rocking chairs in terms of quantitative dimensions, such as height, length, weight, cost, age, color, etc. And we might pose more abstract dimensions or attributes in terms of which we might describe more subtle or qualitative ranges of variation. These might include such variables as "beauty", "character", or "workmanship".

In 1987, I first conceptualized an intuitive proposition which was to guide much of my work that followed. Put simply: "All concepts are made out of dimensions -- and dimensions themselves are made out of dimensions." It is this recursive hypothesis that has led to most of what I have discovered about conceptual dimensionality, and which forms the ontological foundation of my experimental chain of definitions.

1.2. A Rosetta Stone for Cognitive Science?

THE PURPOSE of this present paper is simply to provide an initial overview of this work, to generally describe the methods and principles of this way of looking at conceptual structure, and to provoke some discussion of the controversial and subtle points raised by these propositions. The algebraic consequences of these ideas are complex and detailed, and this paper is only intended as a general survey of the major themes.

As it stands today, Synthetic Dimensionality is not a working computer program or language, and has not been defined in terms of any practical application, such as a database management system, or natural language interpreter. Somewhat like Marvin Minsky's *Society of Mind*, it is merely a body of philosophical and algebraic ideas. But it seems to me that the discovery (or hypothesis) that all conceptual structure can be perfectly modeled in terms of a single linear algebraic primitive element almost certainly has powerful and far-reaching consequences. If I am correct, and it is true that all conceptual structure can be defined in these extremely simple and elegantly recursive terms, I would suppose that the possible applications for this method would be limitless and extremely potent.

Is Synthetic Dimensionality a "Rosetta Stone" for cognitive science? Until this method has enjoyed substantial expansion and development in terms of practical applications and testing, this question cannot truly be answered in the affirmative. But the algebraic simplicity of this method is tantalizing and intriguing, and unless there are hidden errors in this untested method, it seems likely that the representation of conceptual abstractions in terms of linear dimensions will eventually become a powerful and effective tool in our high-speed information processing environment.

2. A Classification of Dimensions

A "DIMENSION" is usually thought of as describing the continuous variation of some numeric variable, and the values of the dimension are the numeric values taken by the variable as it changes, defined as multiples of the "units" of the dimension. "Dimension" and "variable" are almost identical concepts, and a classification of types of variables can be understood as a classification of types of dimensions. Variables (or dimensions) can be classified by the size and properties of their range of values, and by the type of scale which describes them.

The representation of "qualitative" values in terms of dimensions is not a new idea, and "ordinal" variables are common in social and behavioral science. There is an ascending hierarchical relationship between types of variables, which depends on the precision (and the simultaneous dimensionality) with which they are defined. The below classification of dimensions is taken from *Cluster Analysis for Applications*, by Michael R. Anderberg, Academic Press, New York, 1973, p.26:

> A systematic and comprehensive classification of variables [or dimensions] provides a convenient structure for identifying essential differences among [types of] data elements. This section presents a cross-classification of variables based on two familiar descriptive schemes.

2.1. Classification According to Size of the Range Set

FROM a mathematician's point of view, it is natural to distinguish among variables on the basis of the number of variables in the range set, that is, the number of distinct values the variable may assume. To explore this approach adequately it is useful to have the following concepts about counting the number of elements in a set.

1. A set is *finite* if its elements may be put one-to-one correspondence with a sub-set of the positive integers, the latter containing a largest number. Less formally, a set is finite if its elements can be counted and some definite integer is given as the number of elements in the set.

2. A set is *countably infinite* if its elements may be put into one-to-one correspondence with the set of positive integers. The latter set is infinite since there is no largest integer (the claim that some large number, say K, is the largest integer is refuted by exhibiting K + 1, a larger integer). Less formally, the set may be counted, at least in principle, but it would take infinitely long to do so. However,

between any two given elements of the set there is a finite number of other elements in the set.

3. A set is *uncountably infinite* if its elements cannot be put into one-to-one correspondence with the positive integers. Fundamentally, between any two real numbers there are infinitely many real numbers as opposed to the finiteness characterizing an interval in a countably infinite set. Given a starting point in the set, it is not meaningful to speak of "a next number". If the set consists of all real numbers between 1.5 and 2.0, what is "the next number" after 1.5? It is not 1.5000001, since 1.50000009 lies between the two numbers. Indeed, any candidate for "next number" fails since infinitely many numbers may be found between 1.5 and the candidate. Hence, the set is uncountable.

With these concepts, a familiar classification scheme for variables is as follows:

1. A *continuous* variable has an uncountably infinite range set. Typically such a variable may assume any value in an interval (say 1.5 to 2.0) or a collection of such intervals.

2. A *discrete* variable has a finite, or at most a countably infinite range set.

3. A *binary* or *dichotomous* variable is a discrete variable which may take on only two values.

2.2. Classification According to Scale of Measurement

IN THE social and behavioral sciences one frequently encounters a classification of variables based on their scale of measurement. It will be convenient to illustrate this scheme with a variable X and two objects, say A and B, whose scores on X are X(A) and X(B) respectively.

> 1. A *nominal* scale merely distinguishes between classes. That is, with respect to A and B one can only say X(A) = X(B) or X(A) not= X(B).
>
> 2. An *ordinal* scale induces an ordering of the objects. In addition to distinguishing between X(A) = X(B) and X(A) not= X(B), the case of inequality is further refined to distinguish between X(A) > X(B) and X(A) < X(B).
>
> 3. An *interval* scale assigns a meaningful measure of the difference between two objects. One may say not only that X(A) > X(B), but also that A is X(A)-X(B) units different than B.
>
> 4. A *ratio* scale is an interval scale with a meaningful zero point. If X(A) > X(B), then one may say that A is X(A)/X(B) superior to B.
>
> These scale definitions are ordered hierarchically from nominal up to ratio. Each scale embodies all the properties of the scales below it in the ordering. [These scales as listed here are in ascending order] Therefore, by giving up information one may reduce a scale to any lower order scale.
>
> Frequently variables on nominal and ordinal scales are referred to as categorical variables or qualitative variables, often with ambiguity as to whether any order relation exists. For contrast, variables on interval or ratio scales are then referred to as quantitative variables.

It is worth making a special note of Anderberg's comment: "by giving up information one may reduce a scale to any lower order scale." The study of synthetic dimensionality can be understood as a study of the relationship between classes of variables, and involves the precise algebraic analysis of the manner by which information is included or "given up" in the definition of a variable.

3. The Isomorphism of "Dimension" and "Class"

WHEN my pursuit of algebraic epistemology began to converge onto the concept of dimension, I began collecting definitions of dimension from as many sources as possible. I read books on dimensional analysis, as well as in related fields such as cluster analysis and multi-dimensional scaling. And, of course, I also did what I could to work my way through the best literature on fractals, including the original work by Benoit Mandelbrot.

I built lists of alternative definitions of dimension, and for a time I was working with twenty or thirty different ways to define the concept. From mathematics, I brought in the concepts of "set", "class", "cut" (as in "Dedekind cut"), and "variable", as well as "genus and differentia" and "similarity and difference" from the theory of classification, and the concepts of "information", "information structure" and "bit" from computer science.

In my outline processor, I would free-associate my way back and forth between these alternative definitions, looking for every possible linkage and connection, fertilizing my search with the most seminal literature I could find. As this linkage continued to build, and I began to find confirmations of my basic intuitions from a great many alternative points of view, I became increasingly convinced that my basic conviction that "all concepts are built from dimensions" was indeed on the right track.

But as they say in the psychological theory of creativity, there are times when one simply has to abandon one's efforts, and let the creative work bubble in the "unconscious idea processor" of the mind. And my periodic approach to this work took just this form: I would work hard on the project for a week or two, then leave it alone for a couple of months, only to pick it up again. And as I did this, over the course of a couple of years I found my natural instincts and intuitions seeming to converge to one single central definition of dimension, from which it seemed I could then build all others.

As it stands now, that fundamental and underlying definition takes the following form:

3.1. Algebra of dimensions: fundamental definition

A DIMENSION is an ordered class of values, with the following properties:

1. It is a set of distinct elements.
2. This set of elements have in common one or more similarities.
   - These similarities among the elements are defined in terms of values in dimensions; ie, elements are "similar" to the degree that they have identical values in identical dimensions.
3. These similar elements may have distinguishable differences.
   - These differences are defined in terms of values in dimensions; ie, elements are "different" to the degree that they are defined in non-identical dimensions, or have non- identical values in identical dimensions.
   - If these similar elements do not have distinguishable differences (ie, they are identical), the dimension is a "quantitative dimension" of the type which describes physical measurements, and the elements are the units of measure (such as "feet" or "pounds" or "apples").
4. These distinct elements can be ranked in serial/linear order, according to their values in the dimensions in which their differences are defined.
   - A consequence of this ranking is that each ranked element or object in a dimension/class can be interpreted as a value of the dimension.

This definition has a number of properties or consequences which are worth noting:

- It provides a systematic and precise way to define the somewhat ambiguous concepts of similarity and difference.
- It defines a dimension as a class in such a way as is simultaneously consistent with the ordinary intuitive definition of a dimension (such as length as measured in inches) and the intuitive definition of a class (a set of objects with one or more properties in common). If the elements of the class are identical, the dimension is quantitative, and the elements are the units of measure.

- Both the common properties (the similarities), and the distinguishing differences of the elements of the class are defined in terms of dimensions and values in dimensions.
- This approach allows us to use one highly compact and recursively defined algebraic element (ie, dimension) to both abstractly classify and fully describe any object, to any desired degree of specificity. Clearly, when we wish to exactly describe any object, we give its exact measurements in some set of dimensions. Both abstractly classifying and empirically describing any object in terms of a single algebraic concept is elegant, compact, and convenient.
- A dimension is "recursively compositional" -- which is to say that a dimension, like a fractal, is built out of "self- similar" elements. There is a complementary duality between a value and a dimension. Every value is itself a dimension; every dimension is a value.
- Since a dimension is a class of values, and a value is a dimension, "dimensions are built out of dimensions".
- "Class" and "dimension" are isomorphic concepts: an ordered class is a synthetic dimension, and a synthetic dimension is an ordered class.
- As an ordered class, a dimension is not merely any set of objects which we can group together by common properties. In order for a class to be defined as a dimension, we must be able to order (or "sort") the elements of this class into an unambiguous serial/linear sequential list. A class of elements defined in four different dimensions can be sorted in four different ways, according to the values of the elements in each of these dimensions. (An example is the sorting of files on a floppy disk by a personal computer operating system, according to their values in four descriptive parameters: name, date, size, type.)
- A dimension is thus a (unambiguously sequential) *list* of values.
- Just as any value is itself a dimension, any element or member or value of this list (in this definition, "element", "member" and "value" are equivalent and interchangeable concepts) may itself be either a single undifferentiated "unit", or can be itself another list.
- It is interesting to note that these above two points make the definition of dimension intimately related to the fundamental definitions in the

LISP ("list processing") programming language, oftentimes considered the primary language for artificial intelligence (see Douglas Hofstadter, *Metamagical Themas*, pp 396-454).

The definition of dimension is recursive (ie, "defined in terms of itself") in some profound and subtle ways. Not only is there an equivalence between dimension (a class of values) and an ordered class (a class of abstract objects), but each of the values of the dimension are themselves recursively definable as dimensions. It is this recursion which allows me to argue that not only are "all concepts built from dimensions -- but dimensions themselves are built from dimensions".

This can be illustrated in detail by systematically demonstrating that all elements of this above definition can be defined in terms of dimensions. That is, the concepts of "class", "set", "distinction", "similarity", "difference", "category", "element", "member", "value", "rank", and "unit" can all be defined in terms of dimensions -- as can any other basic concept from epistemology. Additionally, I have found that all the basic "data structures" of computer science and linear algebra can be defined in terms of dimensions. The process begins simply by noting the fact that a dimension can be represented as a row vector. Thus, every row vector in a data structure is a dimension of the structure.

In a "quantitative dimension" such as "length in inches", all the inches are the same. Each inch is exactly identical to every other inch, -- with the one exception that each inch is labeled or identified as a particular numeric multiple; ie, the first inch, the second inch, the third inch, etc. This dimension is thus a scale of values, like a ruler or yardstick, where the values are "one dimensional".

I use the phrase "synthetic dimension" to describe any dimension which involves multi-dimensional (or linearly decomposable) values. A "synthetic" dimension is a range of values, just like any other dimension, but its values are not simply identical units, but are instead "similar" units which nevertheless have some distinguishable difference. In this sense, the concept "synthetic dimension" includes the normal intuitive definition of dimension, but is more general, and is defined at a higher level of abstraction.

A consequence of the above fundamental definition is that any ordered class can be thought of as a dimension. Thus, a "set of tea cups", if the cups can be placed in serial order according to some criteria inherent in their description, becomes a (synthetic) dimension. In this dimension, the unit is "tea cups", and they are ordered or sorted by their value in some criteria of their description, such as height or weight or volume.

Synthetic dimensionality offers a way to not only define or fully characterize and describe all objects in terms of quantitative dimensions, but also defines a consistent way that all abstract features, properties, characteristics, and attributes of any object can be defined as (synthetic) dimensions.

4. Categories Defined by Boundary Values in N Dimensions

THERE are any number of ways to define the meaning of "concept", and from my point of view, the words "concept", "class", and "category" are closely related and almost interchangeable. In the context of modern cognitive science, there are generally three approaches taken to the definition of concepts and categories: the "classical" or Aristotelian, the probabilistic, and the "prototypical", associated with Wittgenstein and Zadeh. A good overview text on cognitive science, such as Howard Gardner's *The Mind's New Science*, can introduce the reader to a full discussion of these approaches. Two other excellent sources are Smith and Medin's *Categories and Concepts*, and John Sowa's *Conceptual Structures: Information Processing in Mind and Machine*.

My dimension-based approach to this subject might be described as "modified classicism". That is, I generally approach the definition of classes and categories in Aristotelian terms, but then adapt these terms to a flexible and adaptive algebraic scheme which I believe incorporates the advantages of the other approaches to categorization. This scheme is intended to retain the clarity and simplicity of the classical approach, while avoiding the limitations which these others methods seek to overcome.

John Sowa characterizes these three basic approaches to conceptual definition. From *Conceptual Structures*, p 16:

> For most of the concepts of everyday life, meaning is determined not by definition, but by family resemblance or a characteristic prototype. In a study of concepts, Smith and Medin (1981) summarized three views on definitions:
>
> 1. Classical. A concept is defined by a genus or supertype and a set of necessary and sufficient conditions that differentiate it from other species of the same genus. This approach was first stated by Aristotle and is still used in formal treatments of mathematics and logic. It is the approach that Wittgenstein presented most vigorously in his early philosophy, but rejected in his later writings.
>
> 2. Probabilistic. A concept is defined by a collection of features and everything that has a preponderance of those features is an instance of that concept. This is the position taken by J. S. Mill. It is also the basis for the modern techniques of cluster analysis.
>
> 3. Prototype. A concept is defined by an example or prototype. An object is an instance of a concept c if it resembles the characteristic prototype of c more closely than the prototypes of concepts other than c. This is the position taken by Whewell and is closely related to Wittgenstein's notion of family resemblances.
>
> In fuzzy set theory, Zadeh (1974) tried to formalize the probabilistic point of view. His related theory of fuzzy logic extends uncertainty to every step of reasoning. In prototype theory, however, judgments are made in a state of uncertainty, but once a plant is classified as a member of the rose family, further reasoning is done with discrete logic. Fuzzy set theory has important applications to pattern recognition, but fuzzy logic is problematical. As an adaptation of the classical scheme, I approach the definition of categories and concepts as follows: a category is defined by boundary values (lower and upper) in n (any number) of simultaneous dimensions. Common and distinguishing properties of any member of the category are

> defined by values in dimensions. If an object is characterized by values which are within these boundary values in all of these simultaneous dimensions, the object is "in" that class or category. If one of the values that characterize the description of the object is not within this boundary value range, the object is not in the category.

This method of defining categories requires some clarification, and involves defining exactly what is meant by a dimension. Several initial points should be considered, and each of these requires some clarifying discussion:

1. Any object (whether abstract or concrete) can be symbolically represented (or "modeled") by a set of values in a set of dimensions. In a database, we might represent a category such as "employee" by several dimensions or ranges of value, such as age, education, marital status, seniority, skill level, or any other descriptors which we find useful.

2. The choice of boundary values in categorization involves "drawing a line" along some possibly arbitrary range of values. At what point along a hypothetical continuum does the value "red" become "pink"? This cut-off point or boundary value is defined stipulatively and possibly ad hoc, and not necessarily in terms of some apriori or necessarily system-wide scheme.

3. Any characteristic, feature, attribute or property of an object can be defined as a (synthetic) dimension of the object. If the characteristic or feature is defined in "qualitative" terms, it may have to be linearly factored into a set of quantitative dimensions. This is an important aspect of the doctrine of synthetic dimensionality.

4. The (top-down and linear) factoring of abstract or "qualitative" concepts (or dimensions/attributes) is ad hoc, arbitrary, free-form, and stipulative, and is custom-tailored by a speaker to the exact requirements of usage in an immediate context. It is this principle which enables us to overcome a rigidity in definition which requires some word or concept to "always mean the same thing". I argue that in the context of actual usage, the potential meaning of any word or concept is stipulatively defined ("dimensioned") by the speaker in a way that is fitted to the needs of the immediate moment, and this definition involves a descending hierarchical cascade of increasingly specific linear factors which are initially implicit and unspoken, but can be made

explicit, depending on the need for a higher level of precision.

Thus, the dimension-based approach to defining the meaning of a concept is entirely stipulative (ie, is defined by the user of the concept to suit his/her purposes), and involves two major degrees of freedom: 1) the choice of which dimensions compose the concept, and 2) the choice of boundary values in those dimensions. This approach to definition is fundamentally distinguished from approaches which presume that there is some "best" or "correct" single definition, howsoever conceived. In this context, word meanings are taken from a loosely defined social pool of approximate commonly held meanings, and given exact context- specific values by the speaker. Thus meaning is in part "conventional", but is precisely shaped by stipulation in the context of usage to a specific exact form, characterized by the specific choice of dimensions of composition, and boundary values in those dimensions.

4.1. What is a "cup"?

THESE principles can be illustrated by example. Let us consider the abstract concept of "cup", and ask whether or not some particular actual object is or is not a cup. My dictionary defines a cup as "a small, open container for beverages, usually bowl-shaped and with a handle."

In our nested hierarchy of abstractions, we note that, in general, a "cup" is a specific type of "container for beverages" (ie, a species of the genus "beverage containers", itself a species of "container"), distinguished from other beverage containers by its specific attributes. In our discussion here, we will show how those attributes and distinctions can be defined by values in dimensions, thus determining what is or is not a cup by a series of boundary value ranges which constrain any object within the class "cup".

The general hierarchical structure of the concept cup, according to this definition, is that it is a type of container. Specifically, it is a "beverage" container. And it is distinguished from other beverage containers by certain specific properties.

In the broader class (genus) "beverage container", we might include the following possible members: buckets, pots, cans, glasses, bottles, bowls, cups, and pans -- and we might omit objects such as plates, trays or vases (on the grounds that their dimensional values do not lie within the boundary value range of "beverage container". Clearly, determining whether or not a "plate" is a "beverage container" involves drawing an arbitrary line or cutoff point in some defining dimension, such as the height of the sides of the plate. At what point (along a dimension) does a concavely curved plate become a large shallow bowl? Or, at what point along some range of variation does a liquid, such as gravy, become a "beverage"? Is "soup" a "beverage" -- and if you put enough water in gravy, does it become "soup"?

These distinctions and definitions are generally constrained by social convention and expectation, but in actual usage are exactly defined by a speaker in an arbitrary and ad hoc way. If we say "the gravy was soupy", we are characterizing the gravy in terms of the dimensions of soup, whatever those may be. Determining these in any detail is entirely up to the speaker.

The dimensionality of the concept "cup", as defined by Webster's, can be diagrammed. This scheme interprets the above definition of the generic class "cup" in terms of values in five dimensions:

Object class: "cup"	
Dim 1: "size?"	Value: "small"
Dim 2: "open or closed?"	Value: "open"
Dim 3: "shape?"	Value: "usually bowl-shaped"
Dim 4: "type of object?"	Value: "beverage container"
Dim 5: "handle?"	Value: "usually"

Fig. 1

Defining dimensions and values in these terms may seem unfamiliar, but these values (and dimensions) can be grounded in the familiar quantitative dimensionality through a process of linear factoring. As per Anderberg's scheme, the values "small", "open", and "usually" are ordinal or nominal, while the values "usually bowl-shaped" and "beverage container" require additional factoring (and this factoring, like all conceptual factoring in this scheme, is top-down, linear, and ad hoc).

It is helpful to note that we assign values all the time in such "non-quantitative terms". We might consider "ordinal" and "interval" values describing temperature in terms of the following scheme:

```
          |-------------------|--------------|--------------|
Nominal:  |   uncomfortable   | comfortable  | uncomfortable|
          |----|----|----|----|----|----|----|----|----|----|
Ordinal:  |very cold|  cold   |cool|mild|warm|hot |very hot |
          |----|----|----|----|----|----|----|----|----|----|
Interval: |10  |20  |30  |40  |50  |60  |70  |80  |90  |100 |
          |----|----|----|----|----|----|----|----|----|----|
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Fig. 2

For the sake of convenience, and because we may be operating within a wide boundary value range with no need to be exact (and also because we may be factoring in other simultaneous dimensions as we do so), in some contexts we may be quite content to define temperature as "very cold", with no need to specify that we are referring to a boundary value range between 10.0000 and 29.9999 degrees. Factoring in additional but perhaps non-explicit dimensions, we might define the temperature value as "uncomfortable" -- the various dimensions of "comfort" including not only air temperature, but perhaps how we are feeling, the clothes we are wearing, the humidity or wind-chill, or any other factor.

Thus, in our determination of what is or is not a "cup", we might be defining the values in the various dimensions which describe a cup as "small", or perhaps "shallow", or "tall", but we could assume that these values implied some type of ordinal-to-interval conversion, as per some appropriate implicit scheme, which, if necessary, we could make explicit.

Linear dimensional factoring of definition values:

"Cup"

Synthetic value	Linear dimensional factors
Type: beverage container	Beverages? Containers?
Shape: bowl-shaped	Bowl shape?
Size: small	In what dimensions?
Top: open	(nominal two-state variable)
Handle: usually	(ordinal variable)

Fig. 3

To fully define a cup as a "beverage container", we may have to fully dimension the concepts of "beverage" and "container". We can do this in the same way. What is a container? What is a beverage? Both of these concepts can be linearly factored into their synthetic dimensions.

The value "bowl-shaped" has a hierarchical decomposition; "bowl-shaped" is a type of "shape". To completely specify the dimensions of this concept, we have to first of all decide what are the dimensions of "shape". Then, we must decide what are the dimensions of "bowls". Clearly, this is a process akin to that of defining "cups", and without some grounding in stipulation, convention, or example, we are caught up in a circular definition (ie, a cup is bowl-shaped, and a bowl is cup-shaped). But if pressed to be exactingly specific, we can avoid this problem simply by descending in levels of abstraction, and defining an exact set of specific quantitative dimensions to characterize a particular object.

My dictionary defines "shape" as "the outline or characteristic surface configuration of a thing". Defining this general definition in terms of quantitative dimensions would not be difficult, though in some cases perhaps rather mechanically complex. One might stipulatively create an ordinal dimension that linearly varies by "degree of concavity", and across this dimension distribute such values as plates (almost no concavity), shallow cups and bowls (some concavity), and tall, thin vases and glasses (high concavity). "Bowl-shaped" can thus be seen as an approximate ordinal boundary value range in some posited dimension of linear variation.

The concept "container" can be linearly factored in the same way. The dictionary defines container as "a thing in which material is held or carried." The broader class that includes "container" is "things", a very broad class indeed, and the specific distinguishing characteristic (differentia), that this object holds or carries materials, still defines a very broad class. By narrowing the type of "material" to that of "beverage", we have narrowed the class considerably.

Thus, to answer our question, is this specific object c a "cup", we have to take its measurements in the dimensions of its definition. Is it a "beverage container"? Is it "small"? Is it "open"? Is it "bowl-shaped"? Does it have a handle?

The answer to each of these questions is a value in a class with its own dimensional specifications. Stipulative boundary values must be assigned to determine the exact meaning of words such as "small" -- or the point at which an object become sufficiently concave to be regarded as "bowl-shaped" and not "plate-shaped".

If a specific object is within the boundary value range in each of these defining dimensions, we can define the object as a "cup". If not, we will have to define the object as a member of some other class, such as "plate", or "bowl", or "glass".

We can show each of these properties as a boundary value range in some dimension, whether nominal, ordinal, or interval:

```
                    cup

Size:    -|----|----|----|----|    |----|----|----|-> (ordinal)

Shape:   -|----|----|----|----|    |----|----|----|-> (ordinal)

Type:    -|----|----|----|----|    |----|----|----|-> (ordinal)

Handle:  ---------------------|    ---------------|-> (nominal)

Open:    ---------------------|    ---------------|-> (nominal)
```

Fig. 4

Ordinal values on a dimension such as "size" ("small", "large", etc.) can be factored in various ways, such as the following quantitative descriptors:

```
                    cup

Height: --|----|----|----|----|    |----|----|----|->

Width at diameter: -|----|----|    |----|----|----|->

Volume contained: --|----|----|    |----|----|----|->

Ratio of height to width: ----|    |----|----|----|->
```

Fig. 5

In any of these descriptive dimensions, a "cup" is a class of objects within a boundary value range. Any object that is "too wide" or "too narrow", or "too tall" or "too short" is not a cup.

A "cup" is a highly constrained object, defined within an n-dimensional envelope. Objects within that envelope are cups. Objects that are outside the envelope are not cups.

5. Graphic Models of Synthetic Dimensions

THE graphic representation of a synthetic dimension is inherently two-dimensional. Unlike a conventional quantitative dimension, often modeled on a Euclidean straight line, a synthetic dimension has "width" or "thickness". This is because the elements being characterized by the dimension are being described in two simultaneous dimensions -- their difference and their similarity -- which we can algebraically characterize in the X and Y axes.

As per the fundamental definition, a synthetic dimension is an ordered class of distinct elements, where these elements can be interpreted as the values of the dimension. These elements have one or more similarities or common properties, these common properties being defined as a common boundary value range in a common dimension. That is to say, all the elements of the class (the values of the synthetic dimension) share a boundary value range in at least one common dimension.

But the elements of the class (values of the dimension) are also distinguished from one another by taking differing values (occupying different boundary value ranges) within another common dimension.

We can thus graphically represent a synthetic dimension as a row vector, or linear/sequential list of adjacent cellular addresses, as follows:

```
|----|----|----|----|----|----|----|----|----|----|----|
| a  | b  | c  | d  | e  | f  | g  | h  | i  | j  | k  | D
|----|----|----|----|----|----|----|----|----|----|----|
```

Fig. 6

where the (a,b,c...) represent the elements of the class (values of the dimension). Let us call this dimension D.

This cellular row-vector structure is a convenient graphic model of a synthetic dimension, and allows us to show how similarity can be defined in the Y axis, and difference in the X axis.

Let us presume that X is some quantitative dimension, such as "height", in which all the elements (a,b,c...) are defined. Thus, these elements can be ordered in the X dimension, according to their values in X.

Thus, our row-vector characterization defines the boundary value range in X for each of the elements (a,b,c...), and the "walls" or "edges" of the address cells define these boundary values, thus:

```
  |----|----|----|----|----|----|----|----|----|----|----|
  | a  | b  | c  | d  | e  | f  | g  | h  | i  | j  | k  |
X--|----|----|----|----|----|----|----|----|----|----|----|-->
```

Fig. 7

The cuts in X defined by the boundary values can be defined thus:

```
  X----|----|----|----|----|----|----|----|----|----|----|-->
```

Fig. 8

These cuts are identical to the "Dedekind cut" which characterizes points on the Real Number Line. They can be assigned numeric values, thus:

```
    a    b    c    d    e    f    g    h    i    j    k
X--|----|----|----|----|----|----|----|----|----|----|----|-->
   0    1    2    3    4    5    6    7    8    9   10   11
```

Fig. 9

Therefore, we say that element a has a boundary value range from 0 to 1, element b a range from 1 to 2, c a range from 2 to 3, and so forth.

As we are diagramming these values here, we are showing the distinctions among the (a,b,c...) as being defined by integer values and evenly distributed, but there is no reason for this to be the case. To show a distinction between the elements in the dimension X, it is enough that none of the elements take the same value (share the same boundary value range).

It is perhaps worth noting that there is no real difference between a "value" and a "bounded range of values", since any value is only accurate to within a specified degree of precision, or number of decimal places. A value defined as 3.2563986 is thus still a bounded value range, since we have not specified what the next decimal place might be (and we can, in fact, understand any "next" decimal place as a "species" of the genus defined by the previous decimal place).

Thus, in the graphic representation of synthetic dimensions, we define the differences among elements in terms of values in the X dimension.

The similarity (or common property) which brings the elements together into a class, on the other hand, is defined as a boundary value range in the Y dimension.

Thus, of our elements (a,b,c...) in the dimension D, we define the horizontal boundaries of the address cells, which they all share in common, as a common boundary value range in Y, as in Figure 10:

```
  Y
6-|
  |
5-|
  |
4-|----|----|----|----|----|----|----|----|----|----|----|
  |a   |b   |c   |d   |e   |f   |g   |h   |i   |j   |k   |
3-|----|----|----|----|----|----|----|----|----|----|----|
  |  1    2    3    4    5    6    7    8    9   10   11  X
2-|
  |
1-|
  |
0-|
```

Fig. 10

Continuing to characterize the elements (a,b,c...) in terms of quantitative (numeric) values, we can say that while the differences among the elements are defined by the differing values in the X dimension, all of these elements share a common bounded value range, from 3 to 4, in Y.

Though we are here numerically defining the values of D, there is no reason why we could not define these values ordinally. Both value ranges, in both the X and Y dimensions, could be ordinal, as well as interval, ratio, or nominal.

Objects in a class can share several common properties, which can be represented synthetically in the Y dimension as a single value, characterized by a single class name or label, such as "cup". The single value "cup" can be one of a linear distribution of "beverage containers", which have been sorted according to linear/quantitative criteria in their definition, such as average height or weight or diameter -- or even alphabetical order.

We can thus define the intersection of a class and a "superclass" as the intersection of two synthetic dimensions, as follows, where D* is the superclass "beverage containers", D is the class "cups", and d* is some sub-class. Elements in D can be specific types of cups, such as mugs, tea cups, coffee cups, demitasse, etc., as ordered by some linear/quantitative criteria in their definition.

D* = genus (beverage containers)
D = species (specific cups)
d* = subspecies/sub-class
(a,b,c...) = members of the species, or types of cups
F = cups
X = linear range of variation distinguishing cups
Y = linear range of variation distinguishing beverage containers
(1,2,3...) = either specific cups of type g, or list of n dimensions characterizing all members of (a,b,c...)

```
   Y

   |
 D*|
|----|
| A  |
|----|
| B  |
|----|
| C  |
|----|
| D  |
|----|
| E  |                          d*|
|----|----|----|----|----|----|----|----|----|----|----|
| F  | a  | b  | c  | d  | e  | f  | g  | h  | i  | j  | D
|----|----|----|----|----|----|----|----|----|----|----|-->X
| G  |                         | 1  |
|----|                         |----|
| H  |                         | 2  |
|----|                         |----|-------------
| I  |                     d** | 3  |          ?
|----|                         |----|-------------
| J  |                        | 4  |
|----|                         |----|
                          | 5  |
                          |----|
                          | 6  |
                          |----|         Fig. 11
```

This enables us to show the "inheritance" of features as the orthogonal intersection of two dimensions, rather than the more common representation as a (two-dimensional Aristotelian) tree. The flexibility of this approach comes in part from the fact that each element in these lists is a value on a scale. We can generalize and argue that "similarity is perpendicular to difference".

In response to Hermann Weyl's challenge ("Nobody can say what a variable is."), we might reply by suggesting that the very concept of variable is inherently *at least* two-dimensional, since the data elements (the values of the variable at different times t) all have at least one something in common (they are all values of "the same thing".) Weyl's challenge is at least partly based on the question, "If a variable is constantly changing, how can we say it is always the same thing?" In other words, how does a variable retain a consistent time-invariant identity, while at the same time undergoing state transformations, or taking different possible values? The answer is: a variable is a synthetic data structure, in which, simultaneously, something changes and something remains the same. That alone requires at least two dimensions to describe. If we factor in the problem of concretely representing this data structure in some symbolic medium (such as a computer or a piece of paper), we must factor in any number of additional dimensions of representation...

This discussion only begins to characterize the algebraic consequences of defining synthetic dimensions. There are hundreds or thousands of implications of these definitions, which can and probably should be worked out in the context of a specific applications-oriented engineering and research environment.

6. Ad Hoc Top-Down Decomposition

ANY synthetic value (ie, any concept defined at a level of abstraction above the quantitative) can be linearly factored into its constituent dimensionality. As we saw, the word "cup" can be defined in five dimensions, and several of these dimensions can and must themselves be factored, in order to ground the definition of "cup" in an exact quantitative description which filters out all the objects in the world that are not cups.

Not all words, of course, can be linearly factored into quantitative dimensions. A word like "beauty" has many meanings, which can vary from context to context and from speaker to speaker, and in most contexts cannot be given an exact quantitative definition. In some highly specialized situation, there might be some exceptions to this rule, but, in general, the concept "beauty" is defined in highly "qualitative" dimensions which cannot be realistically grounded in quantitative measurement.

This incomplete or partial dimensional cascade, which extends across descending levels of abstraction, but which does not terminate in quantitative measurement, is typical of a broad class of concepts, which are generally described as "intuitive" or "metaphysical", or even "religious". This incomplete decomposition cascade is the reason that there exists a gulf or divide or fundamental separation between the domains of "science" and of "religion". Generally speaking, the concepts of religion, as they are conceived today, simply cannot be mapped to quantitative measurement. Instead, their meaning lies suspended in a highly abstract space that must be interpreted by faith and belief, rather than in terms of testable and reproducible knowledge. A fuller discussion of dimensional decomposition across the Universal Hierarchy of Abstraction can outline these principles in detail.

The below diagram characterizes the hierarchical decomposition of a concept considerably less abstract than "beauty", but still involving some highly qualitative factors. From H.J. Zimmerman, *Fuzzy Sets, Decision Making, and Expert Systems*, Kluwer Academic Publishers, Boston, 1987, p22:

```
                        Conformity with social
                Business      and economic standards
                behavior---------->|
                  |        Economic thinking
        Personality------->|
          |          |        Motivation
          |        Potential--------->|
Credit-        |                    Physical and mental
worthiness------->|                       potential
          |
          |                     Continuity of margin
          |        Liquidity--------->|
        Financial        |        Income minus expenses
        basis------------->|
                  |        Other net property
                Security---------->|
                        Property minus
                        long-term debts
```

Fig. 12

This schematic provides an approximate "quantification" of the abstract concept of "credit-worthiness", as it might be used without explicit definition in some business context.

Some points to consider:

1. This differentiation is arbitrary and stipulative. It is consciously created in one way rather than another, perhaps by a single individual, perhaps by an organization. It is a "rigid hierarchical cascade of meaning assignments", but it *could* be defined in any number of other ways. It is, then, in a sense an *ad hoc* definition, rather than some ontologically-grounded absolute. The entire cascade could be redefined at any time.

2. The last four variables at the lowest level of abstraction (on the far right) can be numerically quantified. The four other variables (associated with "personality") could probably also be numerically quantified, if we were to extend the hierarchical cascade a few more levels, defining what "we mean by" those still rather qualitative descriptors.

3. Any such ("rigid hierarchical") cascade we might create would *also* be arbitrary, stipulative, and ad hoc. We would create it in one way rather than another to suit some purpose -- and not because we had discovered "the ontologically absolute descriptors of personality".

4. "Credit-worthiness" can thus be defined as at least in part a function of boundary values in the four numerically-quantified variables; each of those variables would have some lowest acceptable value (and, presumably, no highest value).

5. If all of our variables were quantified, we could then define an n-dimensional envelope of boundary values. The credit- worthiness of any individual would be defined as "within" that bounded perimeter. If we are willing to "exercise judgment" in defining the boundary values of qualitative dimensions, we can define the n-dimensional envelope in both qualitative and quantitative dimensions.

This diagram illustrates the two major dimensions of ad hoc meaning specification: the choice of which dimensions compose the meaning of a concept, and the choice of boundary values in those dimensions. In the ascertainment of the credit-worthiness of any individual, we would take the "measurements" of that individual, and load them into the bottom level of our (here, eight dimensional) model, and do a synthetic algebraic compilation, which would have at its top level a two-state (dichotomous or binary) dimension, its two values being: 1) is credit-worthy, and 2) is not credit-worthy. In this framework, depending on the assignment of boundary values (how much of any of these factors is "enough"?), and on the descriptive measurements of the individual, this decision would be algebraically determined.

In the dimensional specification of what is or is not a "cup", we are also free to chose boundary values, the choice of which determines whether a particular object is a "cup" or a "bowl" or a "glass" or a "mug" or a "stein" -- or, in fact, a "plate" or a "table" or a "Dalmatian"...

The structure of a categorical decomposition cascade across levels of abstraction can be generalized, as per Figure 13.

LEVELS OF ABSTRACTION CHARACTERIZED BY TYPE OF DIMENSION

```
<-----Nominal--------Ordinal 2-------Ordinal 1-----Quantitative->

                                  |------|------
                       |-------------->|
                       |            |---------|---
             |------------->|
             |           |            |---|---------
             |           |-------------->|
             |                        |-----------|-
Abstract Concept->|
             |
             |                        |-----|-------
             |           |-------------->|
             |           |            |-|-----------
             |------------->|
                       |            |-----|-------
                       |-------------->|
                                  |----------|--
```

Fig. 13

In this diagram, the horizontal lines represent variation in the X axis (ie, categorical differentia), and the vertical lines represent variation in the Y axis (categorical genus). Thus, each differentiation point takes the following form:

```
             Y

           |   |
           |----|
value in Y--->|   |-----------|----------> X
           |----|          ^
           |   |          value in X
```

Fig. 14

At each level of abstraction, an ordinal value is selected from a range of possible values (the genus), and that ordinal value is re-differentiated, either into lower-level ordinal values, or, at the lowest level, into quantitative values. Figure 13 shows a "binary differentiation" at each level (ie, the cascade splits into only two dichotomous parts), but each level could be differentiated into several dimensions. Each differentiation point represents a choice of value (or boundary value range) at a particular level of abstraction. Thus, at each point where the analysis descends a level of abstraction or precision, there is a differentiation similar to that shown in Figure 14, such that the value in X, shown as a "cut" in Fig. 14, becomes a bounded range in *Y, as per Fig. 15:

```
             Y              *Y

           |   |          |   |
           |----|          |----|
value in Y--->|   |-----------|   |------> X
           |----|          |----|
           |   |          |   |
                   |----|
                   |   |
                   |----|
                   |   |---|---------------->*X
                   |----|   ^Value in *X
                     ^
                  Value in X
```

Fig. 15

Thus, the "quantitative" value defined by a "cut" in X shown in Fig. 14 becomes a bounded range in *Y at a lower level of abstraction. This process occurs at each descending level of abstraction, until some "lowest motivated level" of analysis or precision is defined. That might take an ordinal value, or a quantitative value defined in a specific number of decimal points. Since each new decimal point repeats the differentiation of the boundary value range, this same differentiation (Figures 14 and 15) takes place as each new decimal point value is defined.

The nominal (or dichotomous) determination of whether or not a particular object IS or IS NOT a member of a particular abstract category is determined in a bottom up manner, by taking the "measurements" of the actual object at the lowest level of abstraction (quantitative dimensionality). This decision is algebraically determined by the boundary value choices in the decomposition cascade from the abstract concept to its "meaning" in quantitative dimensions.

The assignment of meaning to an abstract concept, on the other hand, takes place "in the opposite direction", as the abstract concept is linearly factored across a series of descending levels into its quantitative dimensions.

All definition and meaning assignment can be characterized in these terms, and a comprehensive theory of conceptual structure, including a classification of types of concepts, can be characterized in terms of an ad hoc linear decomposition cascade across levels of dimensionality.

This method retains the dichotomous clarity of Aristotelian categories (an object either IS or IS NOT a member of a particular class), but overcomes the paralyzing rigidity which results from attempting to design a single universal categorical taxonomy that is applicable to all situations under all circumstances.

Instead, the method of ad hoc top-down decomposition permits a speaker to emphasize just those shades or dimensions of meaning that are pertinent to a particular context, adjusting those meanings by the exact and context-specific assignment of boundary values.

This analytic description of categorical factoring is entirely consistent with observation from empirical psychology, and appears, in fact, to be "the way we actually do it in real life". This algebraically determinate method overcomes the rigidity of the Aristotelian type hierarchy, and the peculiarities and uncertainties of methods based on "fuzzy logic", as well as the indeterminism of methods based on prototypes.

It is true that any top-down decomposition is inherently rigid and inflexible. But in this model, that inflexibility lasts but for a single moment, under the drive of highly specific motivation, as a single act of meaning-specification is undertaken. When that particular act of communication is completed, that particular cascade of exact meaning assignments is discarded.

7. Synthetic Dimension as Universal Primitive

IT CAN be argued that the fundamental intellectual act is the process of "drawing a distinction" -- or of analytically subdividing some previously unanalyzed "whole unit" into at least two constituent sub-elements.

This act takes the abstract form of a "cut", analogous to the "Dedekind cut" used to define continuity in the Real Number Line, as the previously unitary data element or abstract object is divided into at least two elements, and possibly more.

We have shown this cut in the form of lower and upper boundary values in some range of variation, where "cut-off points" in a dimension are defined to separate categories, as in Figure 9.

But in our indefinitely decomposable (ie, "potentially bottomless") recursive scheme, we are defining the boundary value cuts between classes in terms of dimensional values which themselves may possess a potential "width" in an orthogonal dimension. Thus, every dimension is itself a cut in some "higher" dimension, of which it becomes a sub-class or sub-species.

In the quantitative characterization in Figure 9, the elements (a,b,c...) are being numerically characterized in terms of multiples ("units") of X, but we have not specified the dimensionality of X, which could be linear/quantitative, or multi-dimensional/qualitative.

The generic intersection of D* and D, as per Figure 16, is simply that of a cut defined on a dimension, a boundary value range assigned in Y as common to all the elements (a,b,c...) in X.

```
     Y

    D* |
   |----|
   | A  |
   |----|
   | B  |
   |----|----|----|----|----|----|----|----|----|----|----|
   | C  | a  | b  | c  | d  | e  | f  | g  | h  | i  | j  | D
   |----|----|----|----|----|----|----|----|----|----|----|-->X
   | D  |
   |----|
   | E  |
   |----|
```

Fig. 16

This definition of dimension is recursive. That is, a dimension is defined as an ordered class of dimensions -- as possibly constrained by some "lowest actual level of analysis", where we might find at the bottom of our decomposition cascade some atomic "undifferentiated unit" or "modular part" which we subject to no further decomposition.

As shown in Fig. 13, the decomposition cascade of an abstract concept terminates thus at some "lowest motivated level", usually as defined in some type of quantitative dimensionality.

But the exact specification of this value involves defining a boundary value range, which may be a function of the accuracy of the process of measurement. A "one dimensional" quantitative dimension is analogous to the "real number line" or "continuum", generally regarded as the foundation of mathematics. But the process of actual measurement always involves some degree of error, and actual measurement is always accurate to some specific number of decimal places.

Each new level of decimal place can be understood to represent a new value assignment in a range of possible values (ie, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9). That range of values is a synthetic dimension, with the common factor or "similarity" being that these values all represent differentiations of the previous decimal place.

Thus, a number such as 3.25634 can be understood to define a "genus" with

10 potential differentiations. Whether or not these differentiations are "invoked" or called upon depends upon the motivation involved in determining that level of analysis. Is this level of accuracy required for some reason? Is the rest of the analytic process at the same level of accuracy? Is the measurement process actually this accurate? At some lowest motivated level of decomposition, the analytic process stops, and the value assigned to the variable is perceived as "accurate enough for this application".

7.1. Analog-to-Digital Conversion at the Lowest Level of Analysis

The discrete decimal-place structure of numbers means that the process of measurement involves an "analog-to-digital conversion", as the measurement of the "actual object" is taken by analog means (comparison with a template), and converted to decimal (digital) format, naturally incurring an undefined "round-off error". In general, this process takes the following form, where Y = last decimal place of measurement on some template (unit of measure):

```
 Y**   Y*  Y
----|----|----|-      Y**, Y*, and Y are decimal places.
  |   | 9 |
  |   |----|-       "6" and "3" are sample values, and could
  |   | 8 |        be any value in the range 0-9.
  |   |----|-
  |   | 7 |        This represents the decimal series .63?
  |   |----|-       with the location of the decimal point
  |   | 6 |        left unspecified, and the value "4"
  |   |----|-       taking the last (?) decimal place.
 6 | 3 | 5 |
  |   |----|-
  |   | 4 | -----
  |   |----|-  |
  |   | 3 |   |
  |   |----|-  |
  |   | 2 |   |
  |   |----|-  |
  |   | 1 |   |
  |   |----|-  |
```

```
   |   | 0 |   |
----|----|----|----|---
 ^    ^    ^       ^
 |    |    |       Actual object being measured
 |    |    Lowest calibration of template or "measuring stick"
 |    Second lowest level of calibration
 Third lowest level of calibration
```

Fig. 17

In Figure 17, as we have drawn it here, we show the "actual height" of the object being measured as somewhere around .6343, but since our "measuring stick" is only accurate to the values .004 or .005, we would probably assign the value .004 to this object. This process converts the "continuous analog value of the actual object" to some discrete digital value defined on our template measuring stick, at its lowest level of calibration.

At some lowest motivated level of analysis, where we decide to terminate the decomposition -- and in a quantitative dimension, decide on a specific number places -- we assign value by a process akin to a "Dedekind cut". Value assignment simply takes this form:

```
   |---------------|-----------------------------|------> X
   |           V                        |
lowest possible                    highest possible
  value                            value
```

Fig. 18

where the dimension X is presumed to be continuous (or "potentially continuous"), taking the form of the Real Number Line.

But we must always recognize that the "analog-to-digital conversion" involved in assigning this value means that the value given to V is "actually" a bounded range, as characterized by the unknown "next decimal place" in the series .634?

The implication of this for the study of dimensionality is that "each new decimal place is a new dimension" of measurement, since that potential range of values (0-9) is a synthetic dimension. Thus, the "cut" in the Real Number Line X shown in Figure 18 is potentially a bounded range in Y, as per Figure 14.

7.2. The Dimensional Assembly of All Conceptual Structure

Given that we can thus ground our lowest level of concept definition and specification in the Real Number Line by a sequential process of orthogonal synthetic dimension cuts, we can then reverse the order of definition, and "compositionally assemble" any concept or data structure by ascending the hierarchical order of levels of abstraction.

It so happens that both an alphabet and a vocabulary are synthetic dimensions (ie, they are both a linear/sequential list of distinct elements belonging to a common class that can be linearly ordered -- in this case, by alphabetical order).

Thus, as we ascend the hierarchical structure of abstraction, we assign labels to levels of abstraction we have created by synthetically combining lower-level dimensions and their values. We name these categories with nouns or proper nouns, identified by unique combinations from the elements of the vocabulary dimension (themselves defined by unique combinations from the alphabet dimension).

In this way, abstract ("language-based") symbolic abstraction is "smoothly mapped" into quantitative dimensionality, and from there to the Real Number Line. The implication of this for the study of conceptual structure, whether algebraic or defined in language, is that any conceptual structure can be wholly assembled, in every detail, from the generic primitive element "synthetic dimension", as defined in the fundamental definition.

Synthetic dimensions are the cuts or distinctions themselves, and they also define that which is cut. It is true that an abstract decomposition cascade takes the form of a "cut on a cut on a cut on a cut", in a "self-similar" structure akin to that of a fractal. But it also true that every element of this cascade, including the symbolic elements (or "terminal characters") such as numbers and letters and words, all can be perfectly and exactly defined as synthetic dimensions.

Conclusion

THE concluding argument of this paper is a simple one: all abstract conceptual structure is assembled from synthetic dimensions. Terminological diversity, overlap and inconsistency can be eliminated from the study of conceptual structure by the adoption of this schematic interpretation.

Synthetic dimensionality appears to define the "generic form" of semantic space, and it can probably be argued that every "natural language" is an adaptation and "relabeling" of values and dimensions in this general space, as shaped and individualized by the miscellaneous stresses and influences of cultural evolution and psychological economy.

This paper has outlined only the major aspects of this theory, and has not developed any detailed discussion of specific algebraic implications. These implications, it would seem, can and should receive a full discussion and expansion, in the context of a motivated and applications-oriented engineering and research environment.

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EDITOR'S NOTE:

(*) Bruce Schuman (PO Box 23346, Santa Barbara, CA 93121, U.S.), Founder of United Communities of Spirit (UCS), the most important metaphysical virtual community in the world, have been on the spiritual path since the late 1970's. He have always been interested in the path of transcendence -- what he take to be the way of the prophets and the saints.

“I believe in, and have pursued, the "white fire" initiations of pure energy and absolute self-transformation. (…) For me, electronic networking is a core aspect of my religious beliefs. I don't do this work against the backdrop of my faith -- but as an inherent and deeply-felt part of it. I "believe in" the Internet, in connectivity, in collaboration -- in the idea that no one guru or saint or set of saints and prophets has all the answers for all the people.” - says. UCS: http://www.origin.org/

Ordo Illuminati Ægyptorum

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